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STOCHASTIC BEHAVIOUR OF A TRANSIT SYSTEM HAVING MIXED NETWORKS UNDER HEAD-OF-LINE REPAIR POLICY

By

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ABSTRACT: In this paper, the Author has considered a complex system consisting of two subsystem A and B connected in series. Subsystem A and B are connected in series. Subsystem A consists of N non-identical components in series, while subsystem B consists of four identical components in parallel redundancy.

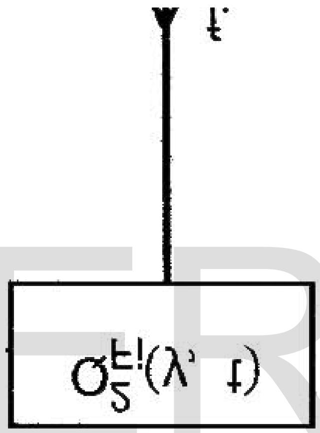
KEYWORDS: Supplimentry variables, Repairable system, Redundancy.

INTRODUCTION

The author has considered a complex system consisting of two subsystem A and B connected in series. Subsystem A and B are connected in series. Subsystem A consists of N-non-identical components in series, while subsystem B consist of four identical components in parallel redundancy. In this the model it is considered that system goes to complete breakdown state if any unit in A fails, or more than three units in the subsystem B are in the failed condition. The system as a whole can also fail from normal efficiency state if there is any human error.

ASSUMPTIONS

(i) Initially, all units are good.
(ii) A failed unit is repaired at a single service channel.
(iii) The parallel subsystem is composed of four identical units, while series subsystem is composed of N non-identical units.
(iv) Failures are statistically independent.
(v) Hardware and human failure rates are constant.
(vi) After repair, units work like new one.
(vii) Repair rates follow general time distribution.
(viii) First come first served repair policy is being adopted.



NOTATIONS:

$$D_x / D_y / D_z / D_t : \frac{\partial}{\partial x} / \frac{\partial}{\partial y} / \frac{\partial}{\partial z} / \frac{\partial}{\partial t}$$

$$f = \sum_{i=1}^{N} f_i$$

No. of units in parallel in subsystem B : 4
No. of units in series in subsystem A : N

$f'/f_i/f_c$: constant failure rates of any unit of B^{th}/i^{th} unit of A/critical human error.

$r_i(x)/S_i(x)$: repair rate of the subsystem A/pdf of transition repair rate.

$r_2(y)/S_2(y)$: repair rate of the subsystem B/pdf of transition repair rate.

$r_c(z)/S_c(z)$: repair rate for critical human error/pdf of transition repair rate.

$\bar{S}_r(S)$: Laplace transform of the pdf of repair time S (t).

$\bar{f}(s)$: Laplace transform of f (t).

$P_N^4(t)$: The probability that at time 't' the system is operating in the state of normal efficiency.

$P_N^3(t)$: The probability that at time't' the system is operating state where I unit of subsystem B has already failed.

$P_N^2(y,t)\Delta$: The probability that at time't' the system is operating in reduced efficiency due to the failure of 2 units of subsystem A, the elapsed repair time lies in the interval (y, y+Δ).

$P_{F_i}^4(x,t)\Delta$: The probability that at time't' the system is in failed state due to the failure of i[th] A and all 4 units of B are in operable state and the elapsed repair time lies in the interval (x, x+Δ).

$P_{F_i}^3(x,t)\Delta$: The probability that at time't' the system is is failed state due to the failure of i[th] A and 3 units of B are operable and the elapsed repair time for A lies in the interval (x, x+Δ); conditioned that no repair is carried out for A.

$P_c(z,t)\Delta$: The probability that at time 't' the system is in failed state due to the critical human error and elapsed repair time for it lies in the interval (z, z + Δ).

$Q_{F_i}^2(y,t)\Delta$` :The probability that at time 't' the system is in failed state due to the failure of i[th] A and 2 units of B, the elapsed repair time of any 2 units of class B lies in the interval ((y, y+ Δ), whereas subsystem A is waiting for repair.

FORMATION OF THE MATHEMATICAL PROBLEM

Viewing the nature of the problem, we obtain the following set of difference-differential equations.

$$(D_t + f + 4f' + f_c)P_N^4(t) = \int_0^\infty P_N^2(y,t)r_2(y)dy + \sum_i \int_0^\infty P_{Fi}^4(x,t)r_i(x)dx + \int_0^\infty P_c(z,t)r_c(z)dz \quad (1)$$

$$(D_t + f + 3f)P_N^3(t) = 4f'P_N^4(t) + \sum_i \int_0^\infty P_{Fi}^3(x,t)r_i(x)dx \quad (2)$$

$$(D_y + D_t + f + r_2(y))P_N^2(y,t) = 0 \quad (3)$$

$$(D_x + D_t + r_i(x))P_{Fi}^4(x,t) = 0 \quad (4)$$

$$(D_x + D_t + r_i(x))P_{Fi}^3(x,t) = 0 \quad (5)$$

$$(D_y + D_t + r_2(y))Q_{Fi}^2(y,t) = f_i P_N^2(y,t) \quad (6)$$

$$(D_z + D_t + r_c(z))P_c(z,t) = 0 \quad (7)$$

These equations are to be solved under the following boundary and initial conditions :

BOUNDARY CONDITIONS

$$P^2_N(0,t) = 3f' P^3_N(t) \quad (8)$$

$$P^4_{Fi}(0,t) = f_i P^4_N(t) + \int_0^\infty Q^2_{Fi}(y,t) r_2(y) dy \quad (9)$$

$$P^3_{Fi}(0,t) = f_i P^3_N(t) \quad (10)$$

$$Q^2_{Fi}(0,t) = 0 \quad (11)$$

$$P_c(0,t) = f_c P^4_N(t) \quad (12)$$

INITIAL CONDITIONS

$P^4_N(0) = 1$ otherwise 0

SOLUTION OF THE PROBLEM

Taking Laplace transform of eqns. (1) through (12) and on solving them one by one, after minor simplification, one may obtain.

$$\overline{P}_N^4(S) = \frac{1}{A(S)} \quad (13)$$

$$\overline{P}_N^3(S) = f'\frac{B(S)}{A(S)} \quad (14)$$

$$\overline{P}_N^2(S) = 3(f')^2 \frac{B(S)}{A(S)} \frac{1-\overline{S}_2(s+f)}{s+f} \quad (15)$$

$$\overline{P}_{Fi}^4(S) = \frac{f_i}{A(S)} \frac{1-\overline{S}_i(s)}{s}\left[1 + \frac{3(f')^2}{f} B(s)\left\{\overline{S}_2(s) - \overline{S}_2(s+f)\right\}\right] \quad (16)$$

$$\overline{P}_{Fi}^3(S) = f_i f' \frac{B(S)}{A(S)} \frac{1-\overline{S}_i(s)}{s} \quad (17)$$

$$\overline{Q}_{Fi}^2(S) = \frac{3f_i(f')^2}{f} \frac{B(S)}{A(S)}\left[\frac{1-\overline{S}_2(s)}{s} - \frac{1-\overline{S}_2(s+f)}{s+f}\right] \quad (18)$$

$$\overline{P}_c(S) = \frac{f_c}{A(S)} \frac{1-\overline{S}_c(s)}{s} \quad (19)$$

EVALUATION OF LAPLACE TRANSFORMS OF UP AND DOWN STATE PROBABILITIES

The Laplace transform of the probabilities that the system is in u.p. (i.e. either good or degraded) and down (i.e. failed) state at time t are as follows :

$$\overline{P}_{up}(s) = \overline{P}_N^4(S) + \overline{P}_N^3(S) + \overline{P}_N^2(S)$$

$$= \frac{1}{A(S)}\left[1 + f'B(s) + 3(f')^2 \frac{1-\overline{S}_2(s+f)}{s+f} B(s)\right] \tag{20}$$

$$\overline{P}_{down}(s) = \overline{P}_c(s) + \sum_{i=1}^{N} \left[\overline{P}^4_{Fi}(s) + \overline{P}^3_{Fi}(s) + \overline{Q}^2_{Fi}(s)\right] \tag{21}$$

It is worth noticing that

$$\overline{P}_{up}(s) + \overline{P}_{down}(s) = \frac{1}{s} \tag{22}$$

PARTICULAR CASES

When repair follows Exponential time distribution : Setting

$$\overline{S}_i(s) = \frac{r_i}{s+r_j}, \overline{S}_2(s) = \frac{r_2}{s+r_2}, \overline{S}_c(s) = \frac{r_c}{s+r_c},\ and\ \overline{S}_2(s+f) = \frac{r_2}{s+f+r_2},$$

in relations (13) through (20), we get

$$\overline{P}^4_N(s) = \frac{1}{D(s)} \tag{23}$$

$$\overline{P}^3_N(s) = f'\frac{E(s)}{D(s)} \tag{24}$$

$$\overline{P}^2_N(s) = 3f'.\frac{E(s)}{D(s)}.\frac{1}{(s+f+r_2)} \tag{25}$$

$$\overline{P}^4_{Fi}(s) = \frac{f_i}{D(s)}.\frac{1}{s+r_i}\left[1 + 3(f')^2.E(s).\frac{r_2}{(s+r_2)(s+f+r_2)}\right] \tag{26}$$

$$\overline{P}^3_{Fi}(S) = f_i f'\frac{E(S)}{D(S)}.\frac{1}{s+r_i} \tag{27}$$

$$\overline{Q}^2_{Fi}(s) = 3f_i(f')^2.\frac{E(s)}{D(s)}.\frac{1}{(s+r_2)(s+f+r_2)} \tag{28}$$

$$\overline{P}_c(s) = \frac{f_c}{D(S)}.\frac{1}{s+r_c} \tag{29}$$

$$\overline{P}_{up} = \frac{1}{D(S)}\left[1 + f'E(s) + \frac{3(f')^2}{(s+f)}.E(s)\right] \tag{30}$$

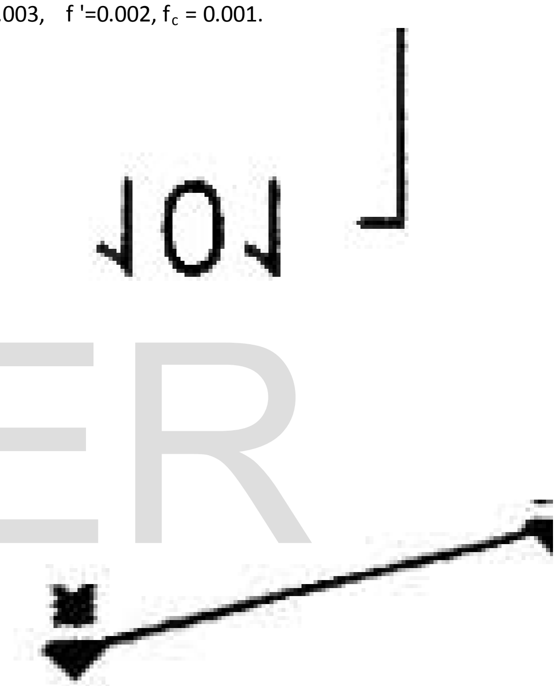
Laplace transform of the probability Pup(t) (i.e the system is in operable state), when there is no repair on the system, is given by

$$\overline{P}_{up}(s) = \frac{1}{(s+f+4f'+f_c)} + \frac{4f'}{(s+f+3f')(s+f+4f'+f_c)} + \frac{12f'}{(s+f)(s+f+3f')(s+f+4f'+f_c)} \tag{31}$$

$$P_{up}(t) = \frac{e^{-ft}}{4f'+f_c}\left[f_c e^{-(4f'+f_c)t+4f'}\right] \tag{32}$$

When system has no repair

Let us consider a physical configuration such that f = 0.003, f '=0.002, f_c = 0.001.



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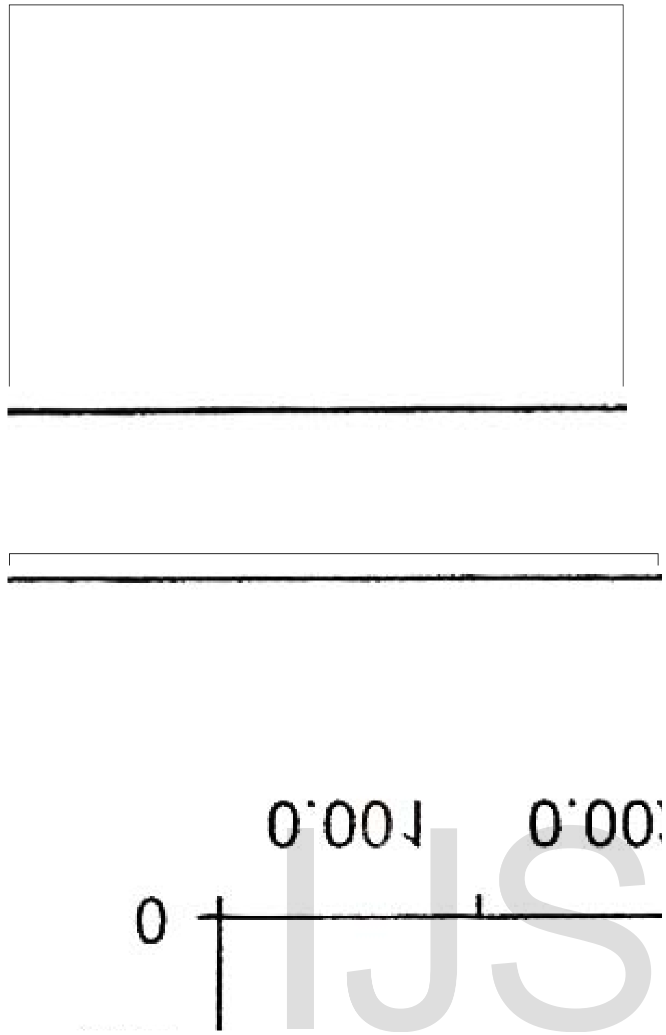

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